4.1. Riemannian curvature tensor with constant sectional curvature. Let (M, g) be a Riemannian manifold with constant sectional curvature $\sec(E) = \kappa \in \mathbb{R}$ for all $E \in G_2(TM)$. Show that

$$R(X, Y)W = \kappa \left(g(Y, W)X - g(X, W)Y \right).$$

4.2. Ricci curvature. Let (M, g) be a 3-dimensional Riemannian manifold. Show the following:

1. The Ricci curvature ric uniquely determines the Riemannian curvature tensor R;
2. If M is an Einstein manifold, then the sectional curvature sec is constant.

4.3. Divergence and Laplacian. Let (M, g) be a Riemannian manifold with Levi-Civita connection D. The *divergence* $\mathrm{div}(Y)$ of a vector field $Y \in \Gamma(M)$ is the contraction of the $(1, 1)$-tensor field $DY : X \mapsto D_X Y$ and the *Laplacian* $\Delta \colon C^\infty(M) \to C^\infty(M)$ is defined by $\Delta f \coloneqq \mathrm{div}(\mathrm{grad}\, f)$. Show that:

1. $\mathrm{div}(fY) = Y(f) + f\mathrm{div}Y$;
2. $\Delta(fg) = f\Delta g + g\Delta f + 2\langle \mathrm{grad}\, f, \mathrm{grad}\, g\rangle$;
3. Compute Δf in local coordinates.